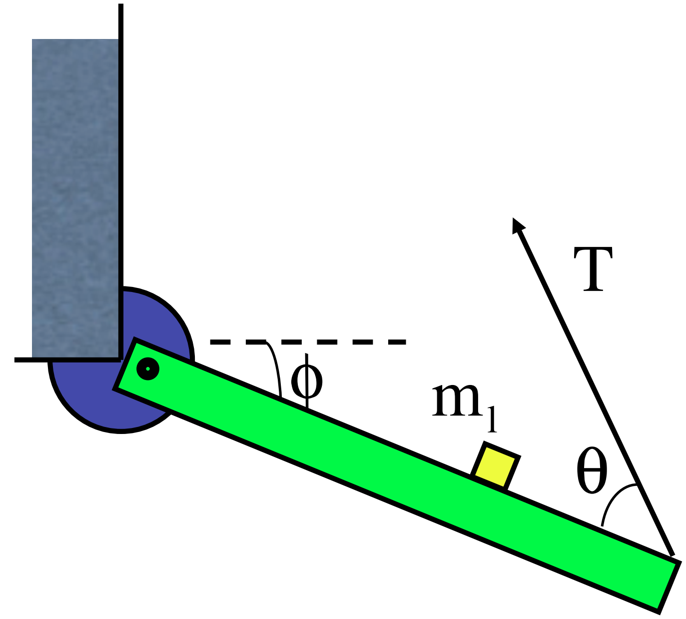
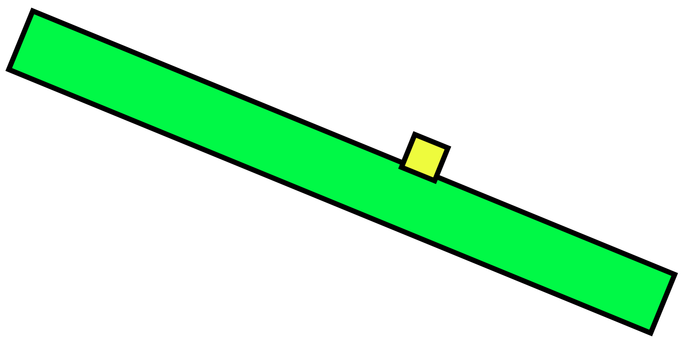
9.) A beam of length "L" is pinned at an angle ϕ with the horizontal. Tension in a rope at an angle θ at the end of the beam (see sketch) keeps it in equilibrium. Glued to the beam is a massive lump a distance "2L/3" units from the pin. Known is:



$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

a.) Draw a f.b.d. for the forces on the beam.



b.) What must the tension in the rope be for equilibrium?

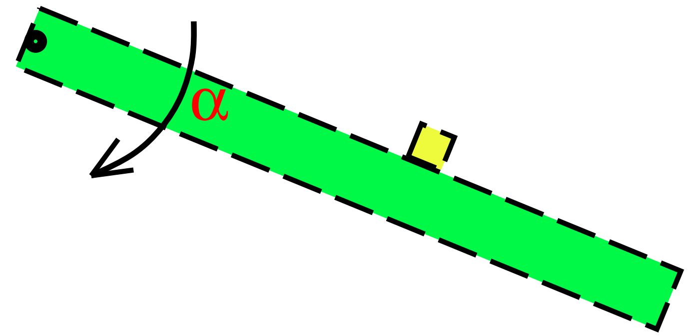
$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

c.) Use the Parallel Axis Theorem to determine "I" of the beam about the pin.

d.) Determine the LUMP'S *moment of inertia* about the pin, and the TOTAL of the system.

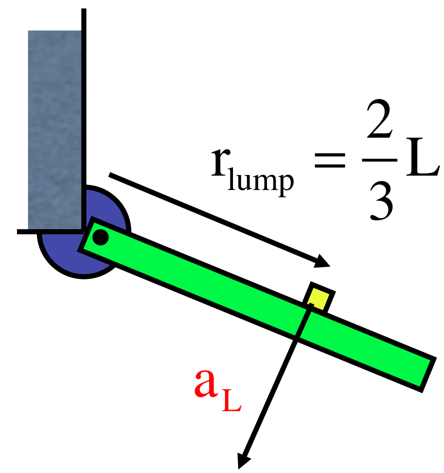
$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

e.) The rope is cut and the beam begins to *angular accelerate* downward. What is the beam's initial angular acceleration?

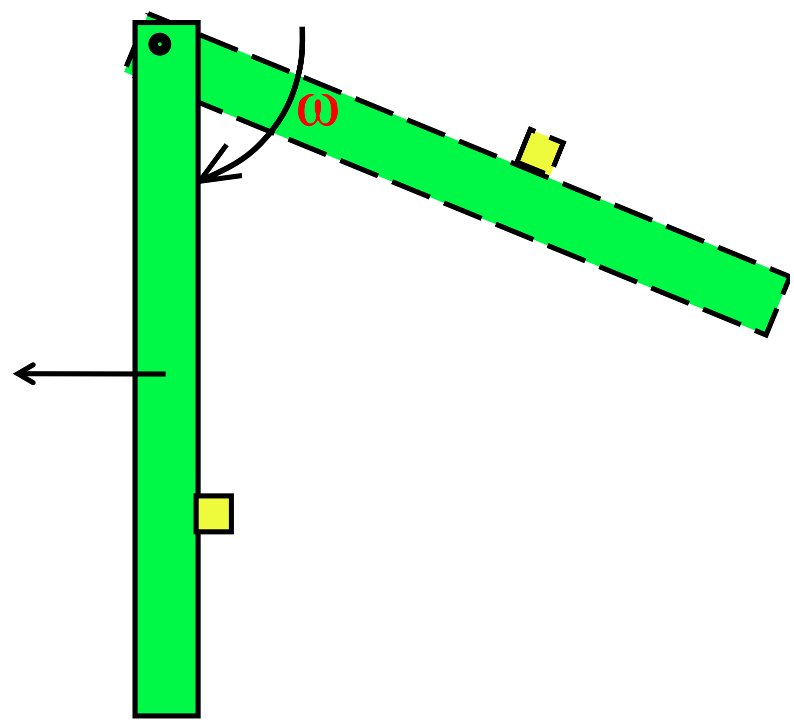


$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

f.) What is the initial acceleration of the lump?



g.) The beam and lump rotate downward. What is their *angular velocity* as they pass through the vertical?

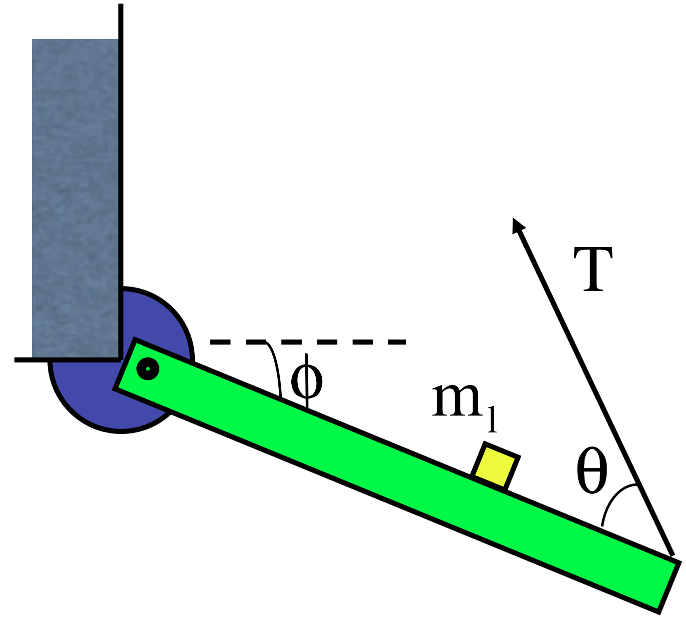


$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

h.) What is the lump's *velocity* as the beam passes through the vertical?

i.) What is the total *angular momentum* about the pin at that point?

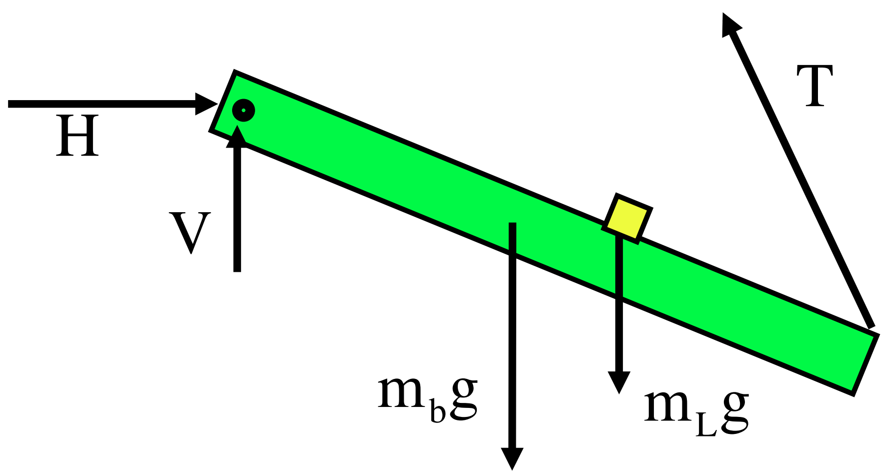
SOLUTION



9.) A beam of length "L" is pinned at an angle ϕ with the horizontal. Tension in a rope at an angle θ at the end of the beam (see sketch) keeps it in equilibrium. Glued to the beam is a massive lump a distance "2L/3" units from the pin. Known is:

$$m_b\,,\ m_L\,,\ L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

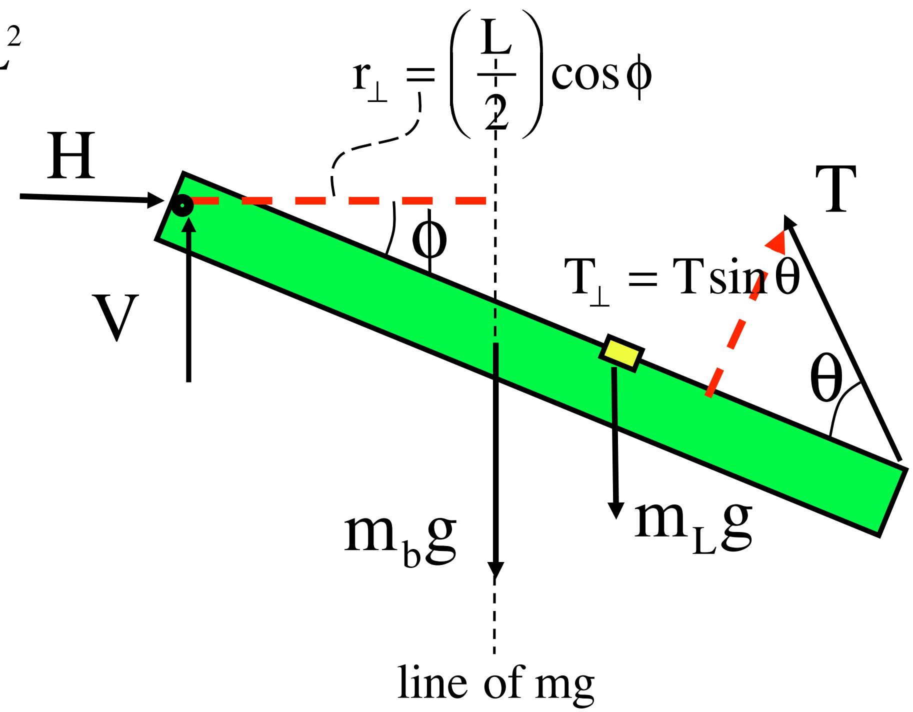
a.) Draw a f.b.d. for the forces on the beam.



m_b, $m_L = .2m_b$, L, g, θ, ϕ and $I_{cm,beam} = \frac{1}{12}m_bL^2$

b.) What must the tension in the rope be for equilibrium?

With r-perpendicular for mg's and F-perpendicular for the tension shown in the sketch, we can use "torque about the pin" to write:



$$\sum \Gamma_{pin}:$$

$$-m_bg\left(\frac{L}{2}\cos\phi\right) - \cancelto{.2m_b}{m_L}g\left(\frac{2L}{3}\cos\phi\right) + (T\sin\theta)L = I_{pin}\cancelto{0}{\alpha}$$

$$\Rightarrow \quad T = \frac{m_bg\left(\frac{\cancel{L}}{2}\cos\phi\right) + .2m_bg\left(\frac{2\cancel{L}}{3}\cos\phi\right)}{\cancel{L}\sin\theta}$$

$$\Rightarrow \quad T = \frac{m_bg\left(\frac{1}{2} + \frac{.4}{3}\right)\cos\phi}{\sin\theta} = \frac{.63m_bg\cos\phi}{\sin\theta}$$

$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

c.) Use the Parallel Axis Theorem to determine "I" of the beam about the pin.

$$I_p = I_{cm} + md^2$$

$$= \frac{1}{12}m_bL^2 + m_b\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3}m_bL^2$$

d.) Determine the LUMP' S *moment of inertia* about the pin, and the TOTAL of the system.

moment of inertia of point-mass lump:

$$I_L = m_L\left(r_{to\ lump}\right)^2$$

$$= \overset{.2m_b}{\cancel{m_L}}\left(\frac{2L}{3}\right)^2$$

$$= \frac{.8}{9}m_bL^2 = .089m_bL^2$$

total moment of inertia:

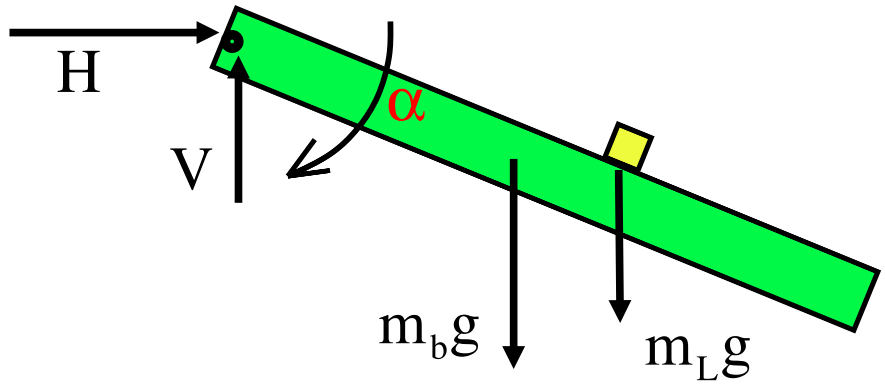
$$I_{pin,tot} = I_L + I_b$$

$$= .089m_bL^2 + \frac{1}{3}m_bL^2$$

$$= .422m_bL^2$$

$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

e.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration?



This is a pure rotation, so we'll sum the torques about the pin:

$$\sum \Gamma_{pin}:$$

$$\cancelto{0}{\Gamma_H} + \cancelto{0}{\Gamma_V} - m_bg\left(\frac{L}{2}\cos\phi\right) - \cancelto{.2m_b}{m_L}g\left(\frac{2L}{3}\cos\phi\right) = -I_{pin,tot}\alpha$$

$$\Rightarrow \quad \alpha = \frac{\cancel{m_b}g\left(\frac{\cancel{L}}{2}\cos\phi\right) + .2\cancel{m_b}g\left(\frac{2\cancel{L}}{3}\cos\phi\right)}{.422\cancel{m_b}L^{\cancel{2}}}$$

$$\Rightarrow \quad \alpha = \frac{\left(\frac{1}{2}+\frac{.4}{3}\right)g\cos\phi}{.422L}$$

$$= \frac{1.5g\cos\phi}{L}$$

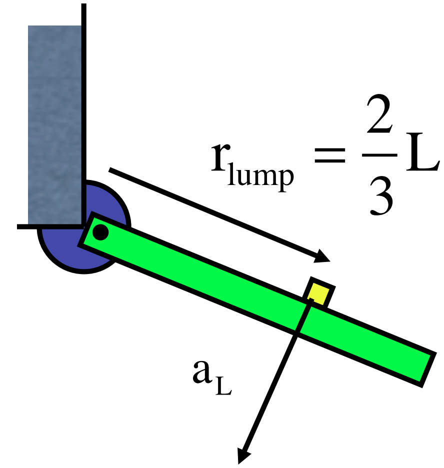
$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

f.) What is the initial acceleration of the lump?

(To the right in red.)

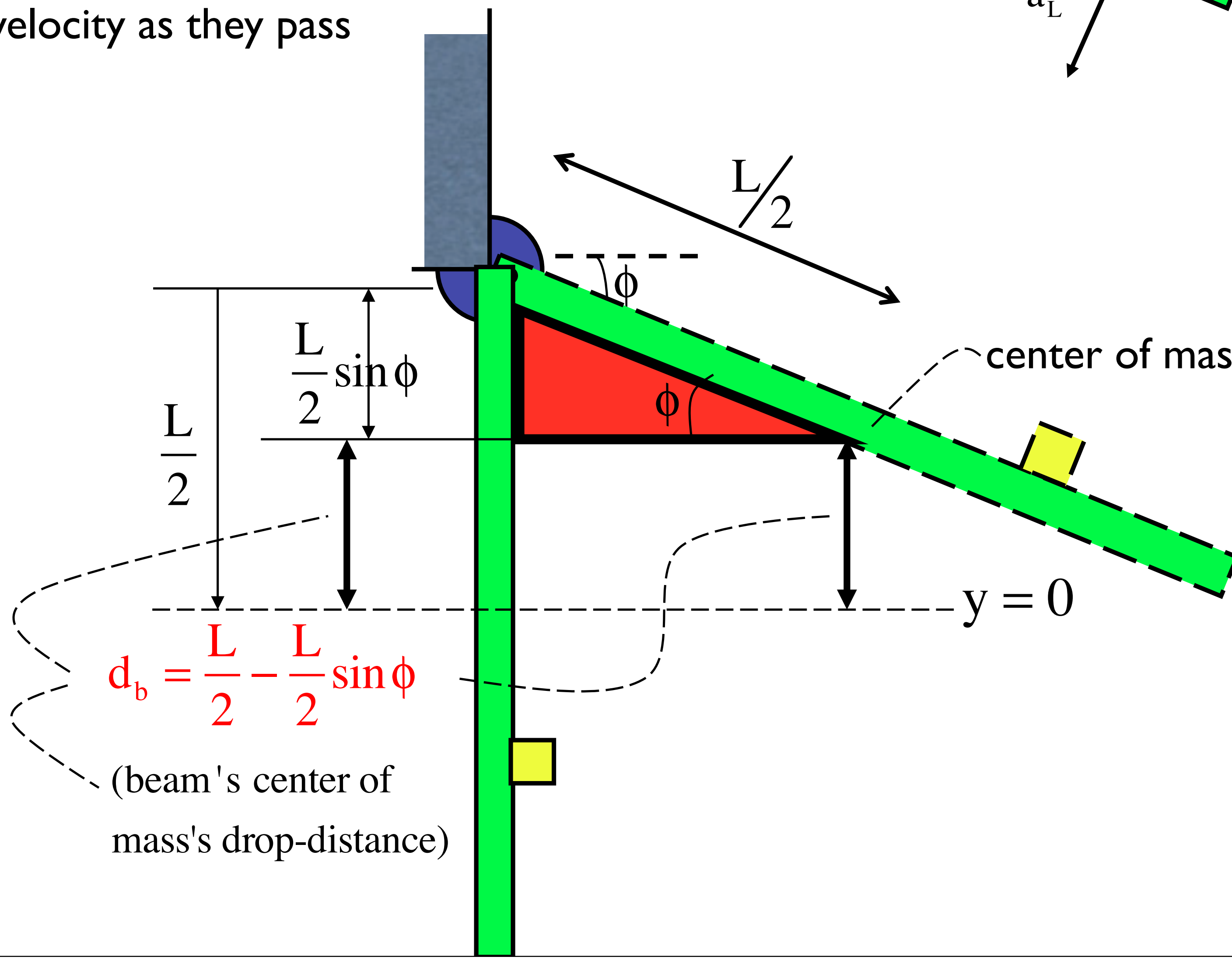
$$a_L = r_{lump}\alpha$$

$$= \left(\frac{2}{3}L\right)\alpha$$



g.) The beam and lump rotate downward. What is their angular velocity as they pass through the vertical?

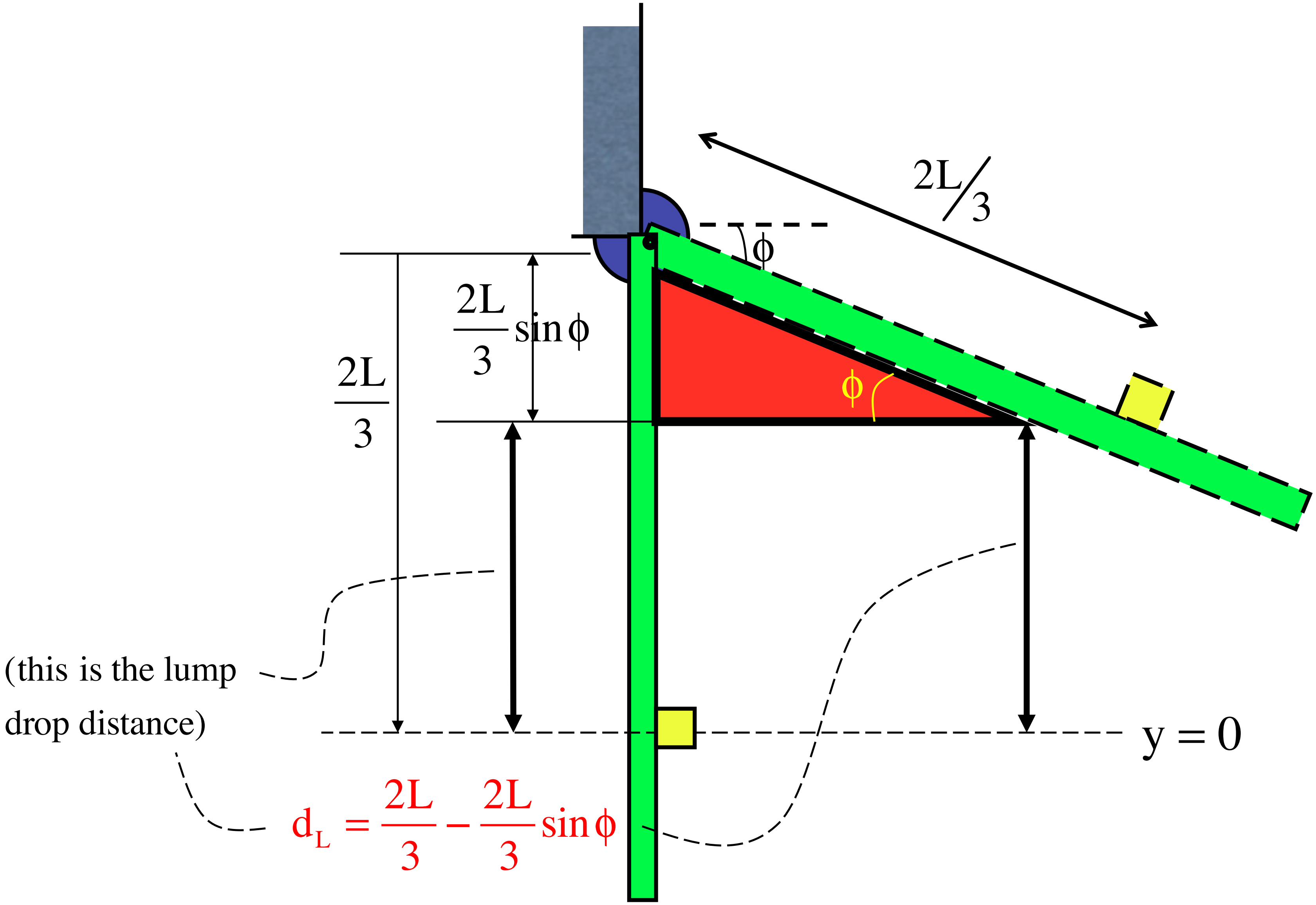
I've identified in the sketch some of the geometry needed to deal with the beam's *center of mess* drop-distance during the fall. The bottom line is in red.



m_b, $m_L = .2m_b$, L, g, θ, ϕ and $I_{cm,beam} = \frac{1}{12}m_bL^2$

g.) (con't.)

As for the *lump*, relevant distances are shown below with the bottom line in red:



$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi$ and $I_{cm,beam} = \frac{1}{12}m_bL^2$

Note that $v_L = r\omega = \left(\frac{2L}{3}\right)\omega$

g.) (con't.)

With the appropriate drop-distances defined on the last two pages, we are ready to use *conservation of energy*. Notice that I *could have* dealt with the final KE as $\frac{1}{2}I_{total}\omega^2$, or I could have treated each piece (the beam and lump) as separate entities. I've done the latter. Also, as a minor side point, note that in solving for the final equation, I divided one of the "L" terms out of the third line, multiplied both sides by 2 and redistributed terms.

treated as a point mass

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + \left[m_b g\, d_b + \overset{.2m_b}{\cancel{m_L}} g\, d_L\right] + 0 = \left[\frac{1}{2}\overset{.2m_b}{\cancel{m_L}}\, v_L{}^2 + \frac{1}{2} I_{b,pin}\,\omega^2\right] + 0$$

$$\Rightarrow \left[m_b g\left(\frac{L}{2} - \frac{L}{2}\sin\phi\right) + .2m_b g\left(\frac{2L}{3} - \frac{2L}{3}\sin\phi\right)\right] = \left[\frac{1}{2}(.2m_b)\left(\frac{2L}{3}\omega\right)^2 + \frac{1}{2}\left(\frac{1}{3}m_bL^2\right)\omega^2\right]$$

$$\Rightarrow \cancel{m_b} g \cancel{L}(.63 - .63\sin\phi) = .211\cancel{m_b}L^{\cancel{2}}\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{2.99g(1-\sin\phi)}{L}}$$

$$m_b,\ m_L = .2m_b,\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_bL^2$$

h.) What is the lump's velocity at that point?

$$v_{lump} = r_{lump}\omega = \left(\frac{2}{3}L\right)\omega$$

i.) What is the total *angular momentum* about the pin at that point? (Yes, I just noticed that I shouldn't be using "L" for both a distance and an angular momentum. I'll be sure not to do that on the test.

$$L_{ang\ mom} = I_{total}\ \omega$$
$$= \left(.422m_bL_b{}^2\right)\omega$$